CASE STUDY

sixteen

Kruskal's Algorithm

case study

OVERVIEW

CS16.1 *Application Overview and Model Development*

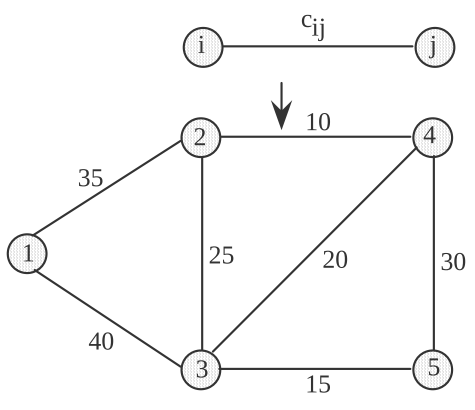
A spanning tree is a connected acyclic sub-graph that spans all of the nodes in a graph. A minimum spanning tree is a spanning tree with the minimum total cost among all spanning trees. We consider the cost of a spanning tree is the sum of the cost of all arcs in the spanning tree. This case study allows the user to specify a network and animate Kruskal's Algorithm to identify a minimum spanning tree.

CS16.1.1 Model Definition and Assumptions

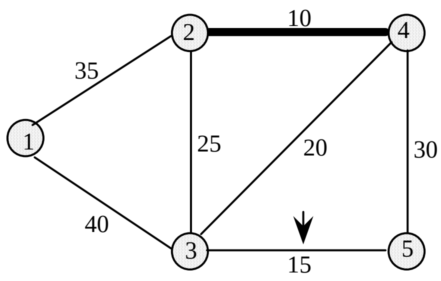
Kruskal's Algorithm is a popular algorithm to determine a minimum spanning tree. This algorithm constructs a spanning tree by adding one arc at a time. The algorithm first sorts all of the arcs in the network in ascending order of their arc costs. Then, it examines each arc in this order one by one and selects it if adding it to the already selected arcs does not create a cycle. When all arcs have been examined, then the selected arcs define a minimum spanning tree.

The intuition behind this algorithm is that if there is an arc cost $c_{ij} > c_{kl}$ for some non-tree arc (k, *l*) and some tree arc (i, j) contained, adding arc (k, *l*) to the tree in place of arc (i, j) gives us a spanning tree with a lower cost. The correctness of Kruskal's algorithm follows from the fact that we discarded each nontree arc (k, *l*) with respect to T^* at some stage because it created a cycle with the arcs already in LIST. But observe that the cost of arc (k, *l*) is greater than or equal to the cost of every arc in that cycle because we examined the arcs in the nondecreasing order of their costs. Therefore, the spanning tree T^* satisfies the path optimality conditions and so it is an optimal tree.

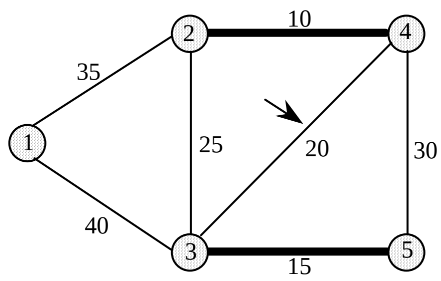
We can illustrate Kruskal's algorithm on a small numerical example. Consider the network shown below. Sorted in the order of their costs, the arcs are (2, 4), (3, 5), (3, 4), (2, 3), (4, 5), (2, 1), and (3, 1). In the first three iterations, the algorithm adds the arcs (2, 4), (3, 5) and (3, 4) to LIST. In the next two iterations, the algorithm examines arcs (2, 3) and (4, 5) and discards them because the addition of each arc to LIST creates a cycle. Then, the algorithm adds arc (2, 1) to LIST and terminates. The last figure below shows the minimum spanning tree.



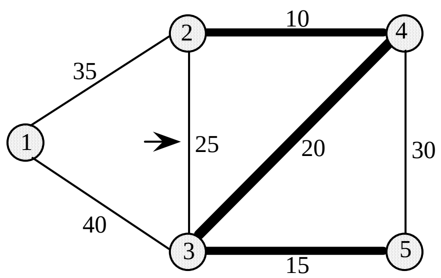
(a)



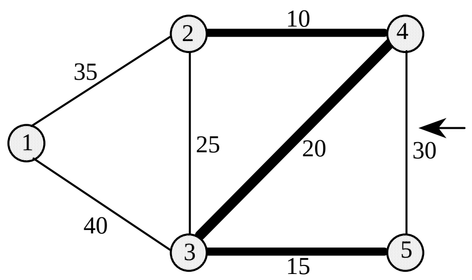
(b)



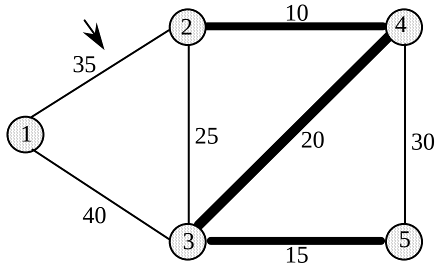
(c)



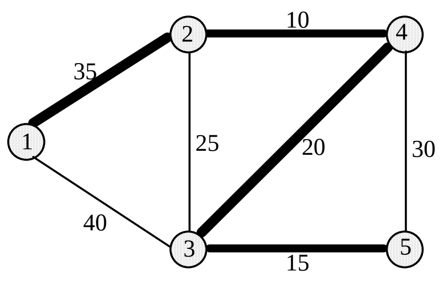
(d)



(e)



(f)



(g)

We assume that the user creates their network and tells us the number of nodes and arcs they have created. We also ask the user to complete a table listing all of the arcs in the network, the nodes they connect, and their arc cost. We use this table to sort the arcs in ascending order of their arc costs. We can then use the node pairs for each listed arc to determine whether or not a cycle would be created if an arc was added to the tree.

For more information about Kruskal's algorithm, please refer to the book *Network Flows: Theory, Algorithm, and Applications* by Ahuja, Magnanti, and Orlin.

CS16.1.2 Input

The input for this application is primarily the network that the user creates.

- Number of nodes and arcs in the network
- Node pairs for each arc
- Arc costs

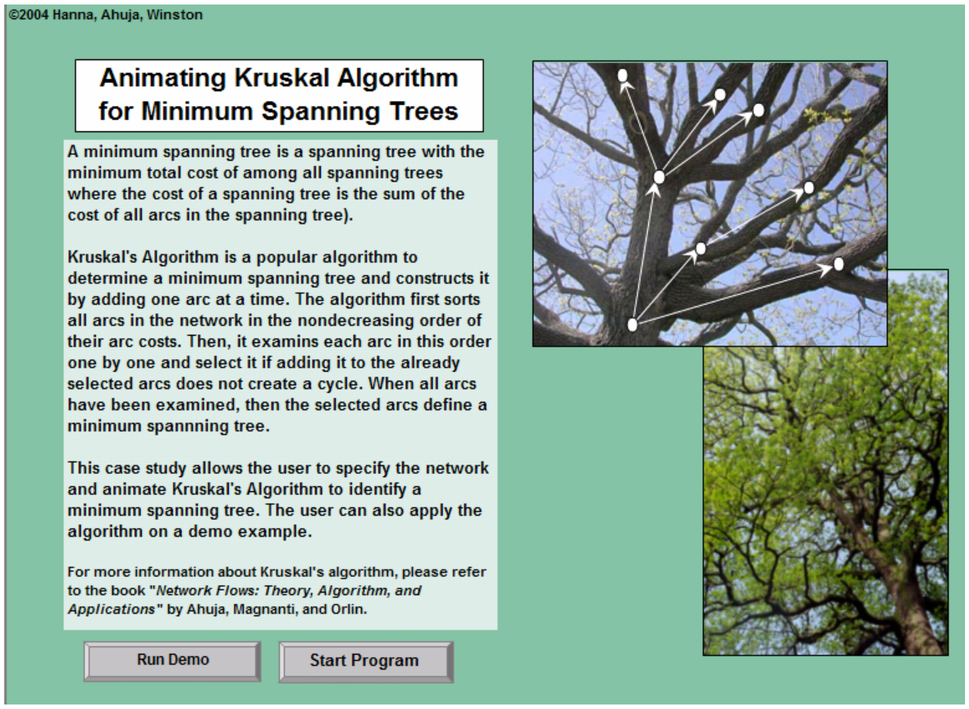
CS16.1.3 Output

The output for this application is the minimum spanning tree.

- Which arcs are in and out of the minimum spanning tree.
- The total spanning tree cost.

CS16.2 Worksheets

We use three worksheets in this application: the welcome sheet, an example sheet, and the network sheet. The welcome sheet has the title and description of the application as well as an image (see Figure CS16.1). There are two buttons on this sheet: "Run Demo" and "Start Program". Both buttons will bring the user to the network sheet.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Animating Kruskal Algorithm for Minimum Spanning Trees

A minimum spanning tree is a spanning tree with the minimum total cost of among all spanning trees where the cost of a spanning tree is the sum of the cost of all arcs in the spanning tree).

Kruskal's Algorithm is a popular algorithm to determine a minimum spanning tree and constructs it by adding one arc at a time. The algorithm first sorts all arcs in the network in the nondecreasing order of their arc costs. Then, it examins each arc in this order one by one and select it if adding it to the already selected arcs does not create a cycle. When all arcs have been examined, then the selected arcs define a minimum spannning tree.

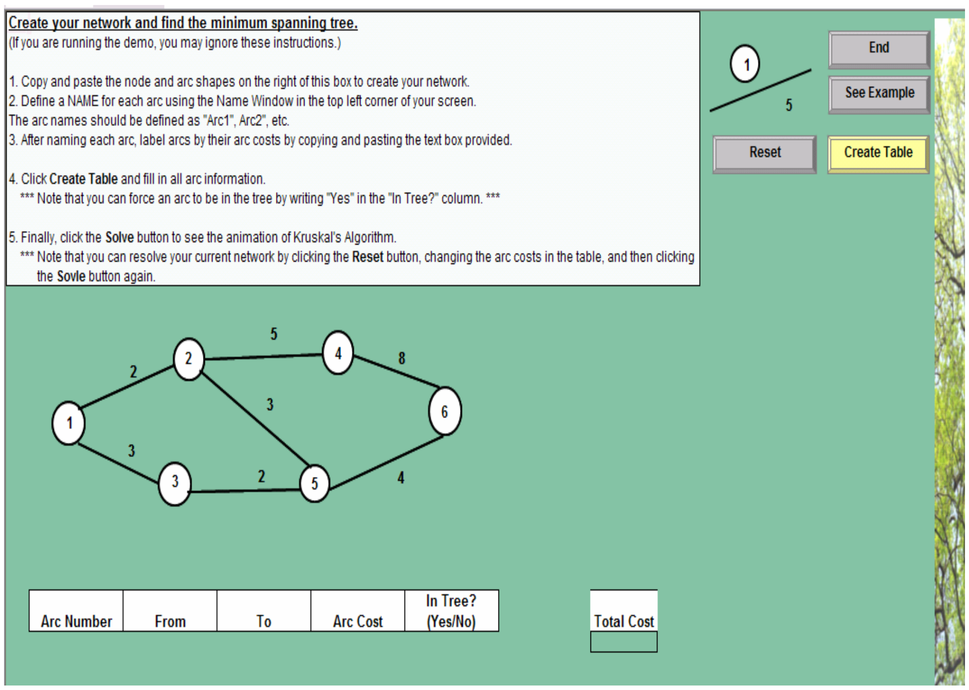
This case study allows the user to specify the network and animate Kruskal's Algorithm to identify a minimum spanning tree. The user can also apply the algorithm on a demo example.

For more information about Kruskal's algorithm, please refer to the book "*Network Flows: Theory, Algorithm, and Applications*" by Ahuja, Magnanti, and Orlin.

Figure CS16.1 The welcome sheet.

The network sheet is the main sheet of the application (see Figure CS16.2). Detailed directions are given to the user at the top of this sheet on how to create their network and how to fill in the table with arc information. The user begins by creating their network. They can copy and paste the node and arc images to create the network in the area above the table. They can also copy and paste the text box to label their arcs with the arc costs. The user must also name each arc using Excel's name window. This visual information is for the animation of Kruskal's algorithm.

When the user is done creating the network, they will press the "Create Table" button. This button will then prompt them with a user form to ask for the number of nodes and arcs in the network they have just created. A table will then be drawn on the sheet for the user to provide the arc information (see Figure CS16.3). This information includes the arc number, node pair that the arc connects, and arc cost. The last column in the table will be used by the application to display to the user which arcs are in and out of the final minimum spanning tree. When the user has completed filling this arc table, they can press the “Solve” button to begin the animation of Kruskal’s algorithm.



Create your network and find the minimum spanning tree.
(If you are running the demo, you may ignore these instructions.)

1. Copy and paste the node and arc shapes on the right of this box to create your network.
2. Define a NAME for each arc using the Name Window in the top left corner of your screen.
The arc names should be defined as "Arc1", Arc2", etc.
3. After naming each arc, label arcs by their arc costs by copying and pasting the text box provided.

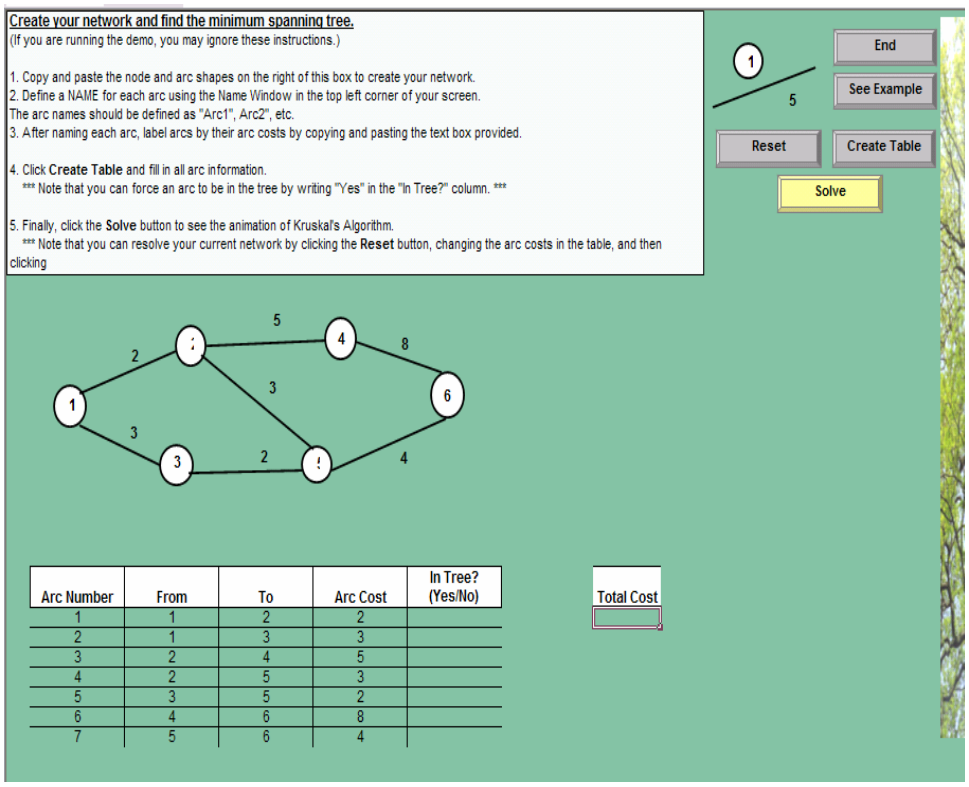
4. Click **Create Table** and fill in all arc information.
*** Note that you can force an arc to be in the tree by writing "Yes" in the "In Tree?" column. ***

5. Finally, click the **Solve** button to see the animation of Kruskal's Algorithm.
*** Note that you can resolve your current network by clicking the **Reset** button, changing the arc costs in the table, and then clicking the **Sovle** button again.

Arc Number	From	To	Arc Cost	In Tree? (Yes/No)

Total Cost

Figure CS16.2 The network sheet after creating the network.



Create your network and find the minimum spanning tree.
(If you are running the demo, you may ignore these instructions.)

1. Copy and paste the node and arc shapes on the right of this box to create your network.
2. Define a NAME for each arc using the Name Window in the top left corner of your screen.
The arc names should be defined as "Arc1", Arc2", etc.
3. After naming each arc, label arcs by their arc costs by copying and pasting the text box provided.

4. Click **Create Table** and fill in all arc information.
*** Note that you can force an arc to be in the tree by writing "Yes" in the "In Tree?" column. ***

5. Finally, click the **Solve** button to see the animation of Kruskal's Algorithm.
*** Note that you can resolve your current network by clicking the **Reset** button, changing the arc costs in the table, and then clicking

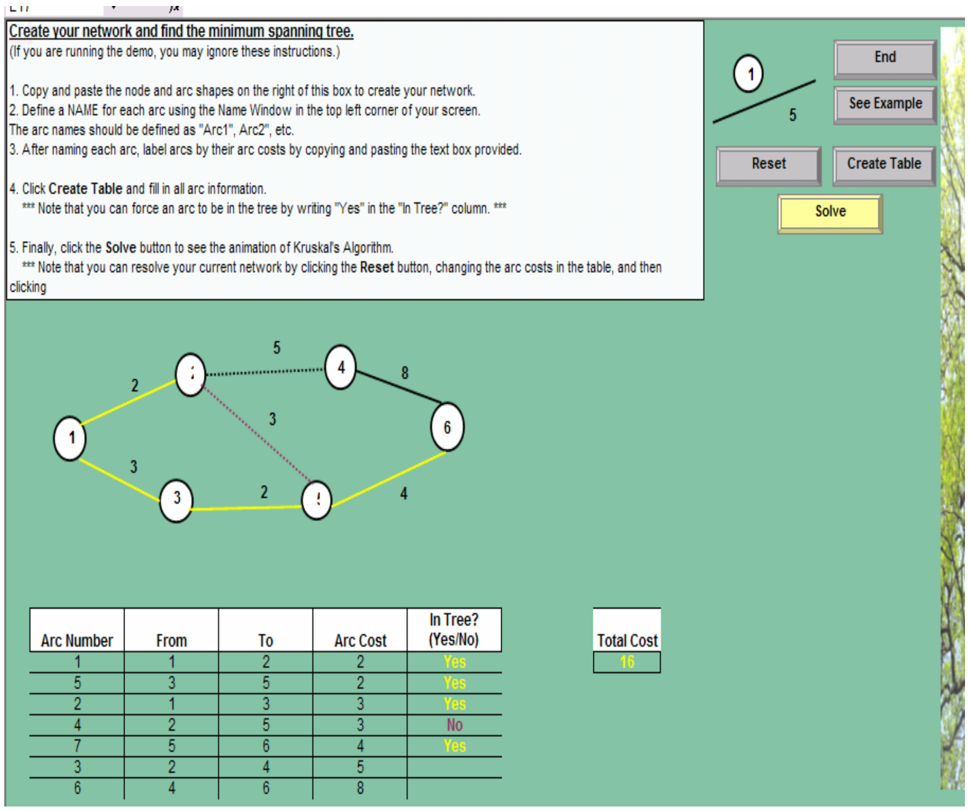
Arc Number	From	To	Arc Cost	In Tree? (Yes/No)
1	1	2	2	
2	1	3	3	
3	2	4	5	
4	2	5	3	
5	3	5	2	
6	4	6	8	
7	5	6	4	

Total Cost

Figure CS16.3 The network sheet after creating the table.

As the creation of the minimum spanning tree is being performed one arc at a time by Kruskal’s algorithm, the user can see which arcs are being evaluated and whether or not the evaluated arc has been put in the tree or kept out. As each arc is evaluated a “Yes” or “No” will be displayed in the arc table to show whether or not the arc is in the minimum spanning tree.

In Figure CS16.4, we show an example network in the middle of the algorithm animation. Here, the first five arcs have been evaluated: the first three are in the tree, the fourth is out of the tree, and the fifth is in the tree. The sixth arc is currently being evaluated. Note that the animation of the arc evaluations in the network corresponds to the order of the rows in the arc table.



Create your network and find the minimum spanning tree.
(If you are running the demo, you may ignore these instructions.)

1. Copy and paste the node and arc shapes on the right of this box to create your network.
2. Define a NAME for each arc using the Name Window in the top left corner of your screen.
The arc names should be defined as "Arc1", Arc2", etc.
3. After naming each arc, label arcs by their arc costs by copying and pasting the text box provided.

4. Click **Create Table** and fill in all arc information.
*** Note that you can force an arc to be in the tree by writing "Yes" in the "In Tree?" column. ***

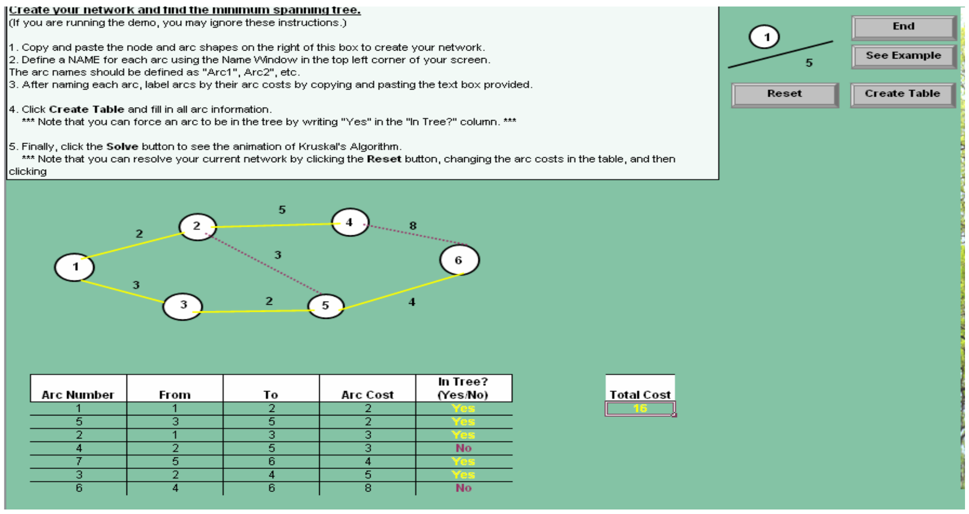
5. Finally, click the **Solve** button to see the animation of Kruskal's Algorithm.
*** Note that you can resolve your current network by clicking the **Reset** button, changing the arc costs in the table, and then clicking

Arc Number	From	To	Arc Cost	In Tree? (Yes/No)
1	1	2	2	Yes
5	3	5	2	Yes
2	1	3	3	Yes
4	2	5	3	No
7	5	6	4	Yes
3	2	4	5	
6	4	6	8	

Total Cost
16

Figure CS16.4 The network sheet during the animation.

The final minimum spanning tree for this example is shown in Figure CS16.5. Once the algorithm animation is complete, the user can see which arcs are in and out of the minimum spanning tree by viewing the colors of the arcs in the network and the last column in the arc table. The total cost of the spanning tree is also displayed. This cost is the sum of all arc costs of the arcs in the minimum spanning tree. The user can now press “Reset” in order to change arc costs and re-solve, or they can exit the application by pressing the “End” button.



Create your network and find the minimum spanning tree.
(If you are running the demo, you may ignore these instructions.)

1. Copy and paste the node and arc shapes on the right of this box to create your network.
2. Define a NAME for each arc using the Name Window in the top left corner of your screen.
The arc names should be defined as "Arc1", Arc2", etc.
3. After naming each arc, label arcs by their arc costs by copying and pasting the text box provided.

4. Click **Create Table** and fill in all arc information.
    *** Note that you can force an arc to be in the tree by writing "Yes" in the "In Tree?" column. ***

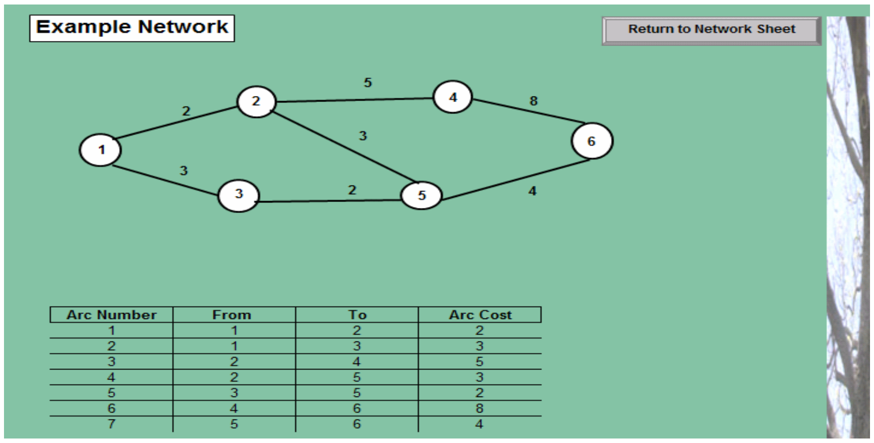
5. Finally, click the **Solve** button to see the animation of Kruskal's Algorithm.
    *** Note that you can resolve your current network by clicking the **Reset** button, changing the arc costs in the table, and then clicking

Arc Number	From	To	Arc Cost	In Tree? (Yes/No)
1	1	2	2	Yes
5	3	5	2	Yes
2	1	3	3	Yes
4	2	5	3	No
7	5	6	4	Yes
3	2	4	5	Yes
6	4	6	8	No

Total Cost
16

Figure CS16.5 The network sheet with the final solution.

The example sheet is available to the user if they press the "See Example" button on the network sheet. The example sheet has an example network and completed arc table (see Figure CS16.6). This sheet is used to guide the user in creating their own network and filling in their own arc table. The user can press the "Return to Network Sheet" button to return to the network sheet. The data on this sheet is also used for running the demo of the application.



Arc Number	From	To	Arc Cost
1	1	2	2
2	1	3	3
3	2	4	5
4	2	5	3
5	3	5	2
6	4	6	8
7	5	6	4

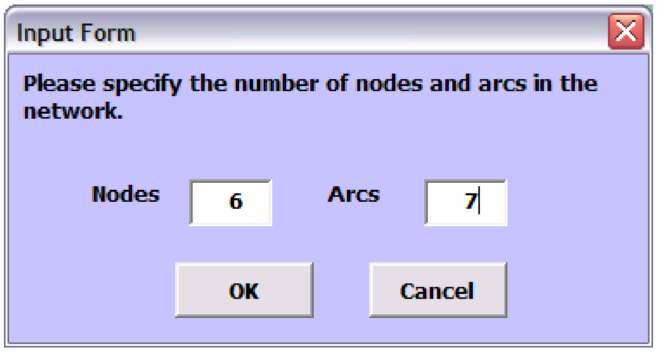
Figure CS16.6 The example sheet.

Summary

Welcome sheet	Application description and "Start" button.
Network sheet	The main sheet in which the user creates their network, completes a table of arc information, and runs the algorithm animation.
Example sheet	An example network and completed arc table are available to user for guidance; this information is also used to run the demo.

CS16.3 User Interface

For this application, we use navigational and functional buttons as well as one user form for the user interface. On the welcome sheet, there are two buttons: "Run Demo" and "Start Program". Both buttons take the user to the network sheet, but if the "Run Demo" button was pushed then the network will already be created. When the user presses the "Create Table" button, the user form appears to prompt them for the number of nodes and arcs in the network they have created (see Figure CS16.7). If the demo is run and the network was already created for the user, then this form is not shown.


Input Form

Please specify the number of nodes and arcs in the network.

Nodes: 6 Arcs: 7

Figure CS16.7 The input form.

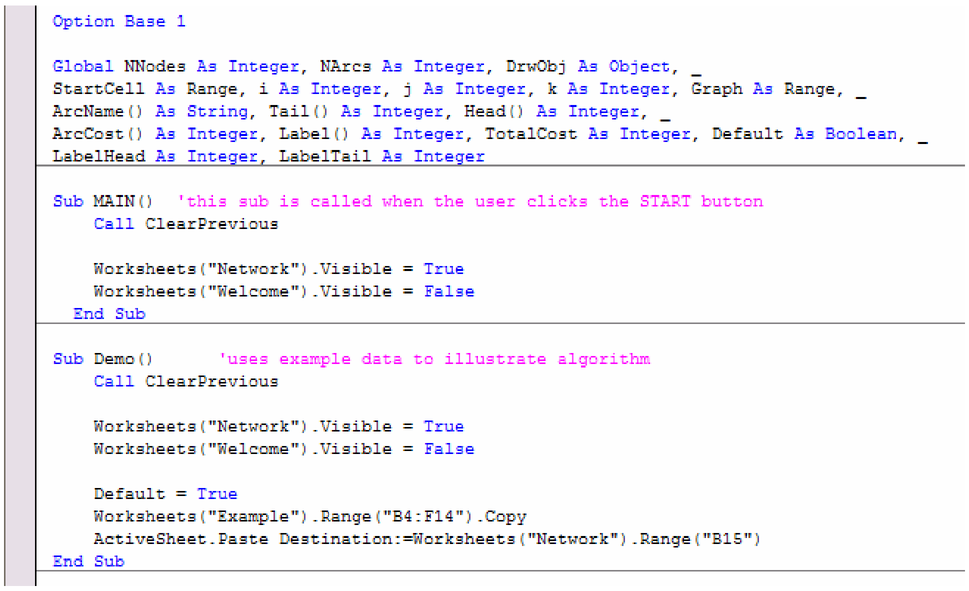
The remainder of the interface includes the functional buttons "Solve" and "Reset" as well as the navigational buttons "See Example", "Return to Network Sheet", and "End". Some formatting is done to highlight the button which should be pressed at the necessary time. This is another clarification tool incorporated in the user interface.

Summary

Navigational Buttons	"Run Demo" and "Start Program" buttons on welcome sheet; "See Example" and "End" buttons on the network sheet; "Return to Network Sheet" on the example sheet.
Functional Buttons	"Create Table", "Solve", and "Reset" buttons on the network sheet.
Input user form	User specifies number of nodes and arcs in the network they have created.

CS16.4 Procedures

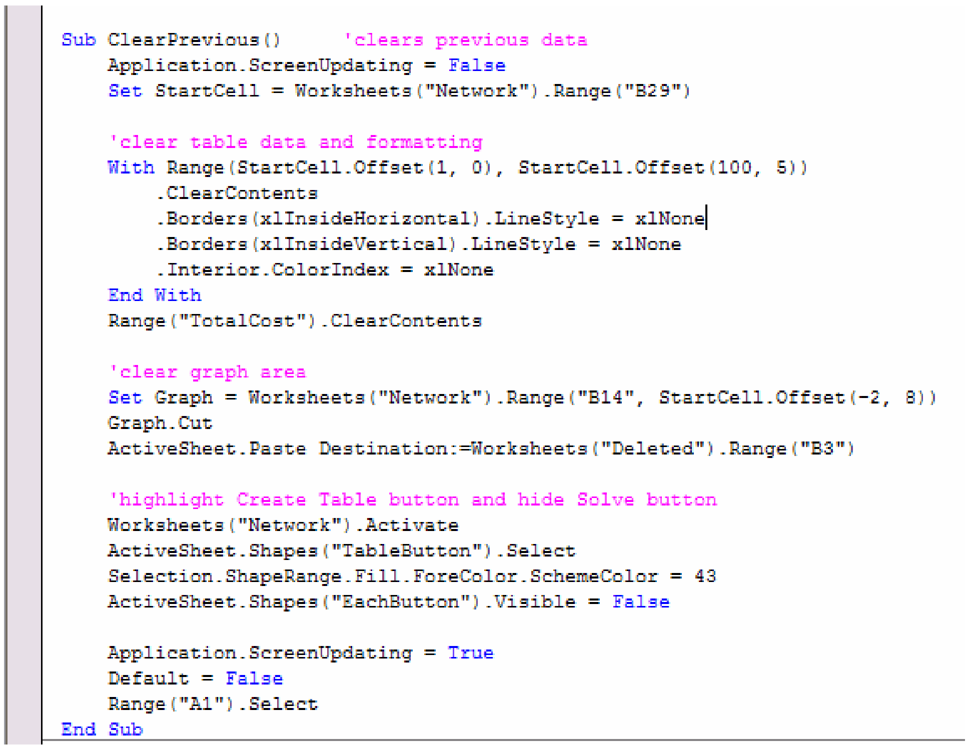
We will now outline the procedures for this application. We begin with the initial sub procedures and variable definitions (see Figure CS16.8). The *Main* procedure, assigned to the "Start Program" button, calls the *ClearPrevious* procedure and takes the user to the network sheet. The *Demo* procedure, assigned to the "Run Demo" button, calls the *ClearPrevious* procedure, takes the user to the network sheet, and copies the network from the example sheet to the network sheet.



```
Option Base 1

Global NNodes As Integer, NArcs As Integer, DrwObj As Object, _
StartCell As Range, i As Integer, j As Integer, k As Integer, Graph As Range, _
ArcName() As String, Tail() As Integer, Head() As Integer, _
ArcCost() As Integer, Label() As Integer, TotalCost As Integer, Default As Boolean, _
LabelHead As Integer, LabelTail As Integer

Sub MAIN()  'this sub is called when the user clicks the START button
    Call ClearPrevious

    Worksheets("Network").Visible = True
    Worksheets("Welcome").Visible = False
  End Sub

Sub Demo()      'uses example data to illustrate algorithm
    Call ClearPrevious

    Worksheets("Network").Visible = True
    Worksheets("Welcome").Visible = False

    Default = True
    Worksheets("Example").Range("B4:F14").Copy
    ActiveSheet.Paste Destination:=Worksheets("Network").Range("B15")
End Sub
```

Figure CS16.8 Variable declarations, *Main* procedure and *Demo* procedure.

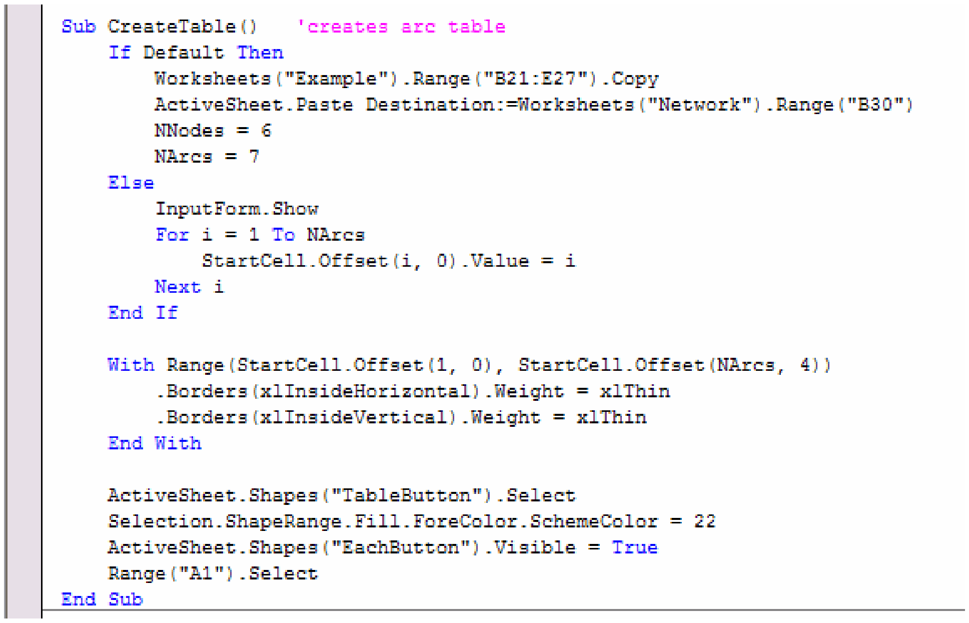
The *ClearPrevious* procedure clears previous data and initializes some variable values (see Figure CS16.9). It also clears the formatting of the arc table and deletes the network images. Some button formatting is also modified here.



```
Sub ClearPrevious()     'clears previous data
    Application.ScreenUpdating = False
    Set StartCell = Worksheets("Network").Range("B29")

    'clear table data and formatting
    With Range(StartCell.Offset(1, 0), StartCell.Offset(100, 5))
        .ClearContents
        .Borders(xlInsideHorizontal).LineStyle = xlNone
        .Borders(xlInsideVertical).LineStyle = xlNone
        .Interior.ColorIndex = xlNone
    End With
    Range("TotalCost").ClearContents

    'clear graph area
    Set Graph = Worksheets("Network").Range("B14", StartCell.Offset(-2, 8))
    Graph.Cut
    ActiveSheet.Paste Destination:=Worksheets("Deleted").Range("B3")

    'highlight Create Table button and hide Solve button
    Worksheets("Network").Activate
    ActiveSheet.Shapes("TableButton").Select
    Selection.ShapeRange.Fill.ForeColor.SchemeColor = 43
    ActiveSheet.Shapes("EachButton").Visible = False

    Application.ScreenUpdating = True
    Default = False
    Range("A1").Select
End Sub
```

Figure CS16.9 The *ClearPrevious* procedure.

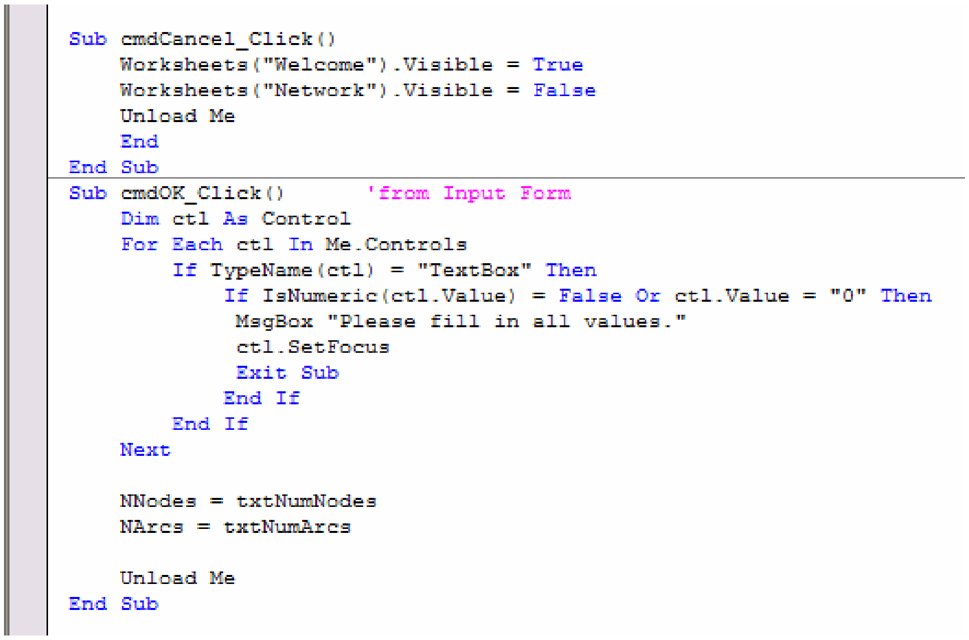
When the user presses the "Create Table" button, the *CreateTable* procedure is called (see Figure CS16.10). If the demo is being run, then the arc table is copied from the example sheet and pasted in the network sheet. However, if the user has created their own network, then the input form will now be shown to receive the values for the number of nodes and arcs in the network. The arc table is then formatted as well as some of the buttons.



```
Sub CreateTable()   'creates arc table
    If Default Then
        Worksheets("Example").Range("B21:E27").Copy
        ActiveSheet.Paste Destination:=Worksheets("Network").Range("B30")
        NNodes = 6
        NArcs = 7
    Else
        InputForm.Show
        For i = 1 To NArcs
            StartCell.Offset(i, 0).Value = i
        Next i
    End If

    With Range(StartCell.Offset(1, 0), StartCell.Offset(NArcs, 4))
        .Borders(xlInsideHorizontal).Weight = xlThin
        .Borders(xlInsideVertical).Weight = xlThin
    End With

    ActiveSheet.Shapes("TableButton").Select
    Selection.ShapeRange.Fill.ForeColor.SchemeColor = 22
    ActiveSheet.Shapes("EachButton").Visible = True
    Range("A1").Select
End Sub
```

Figure CS16.10 The *CreateTable* procedure.

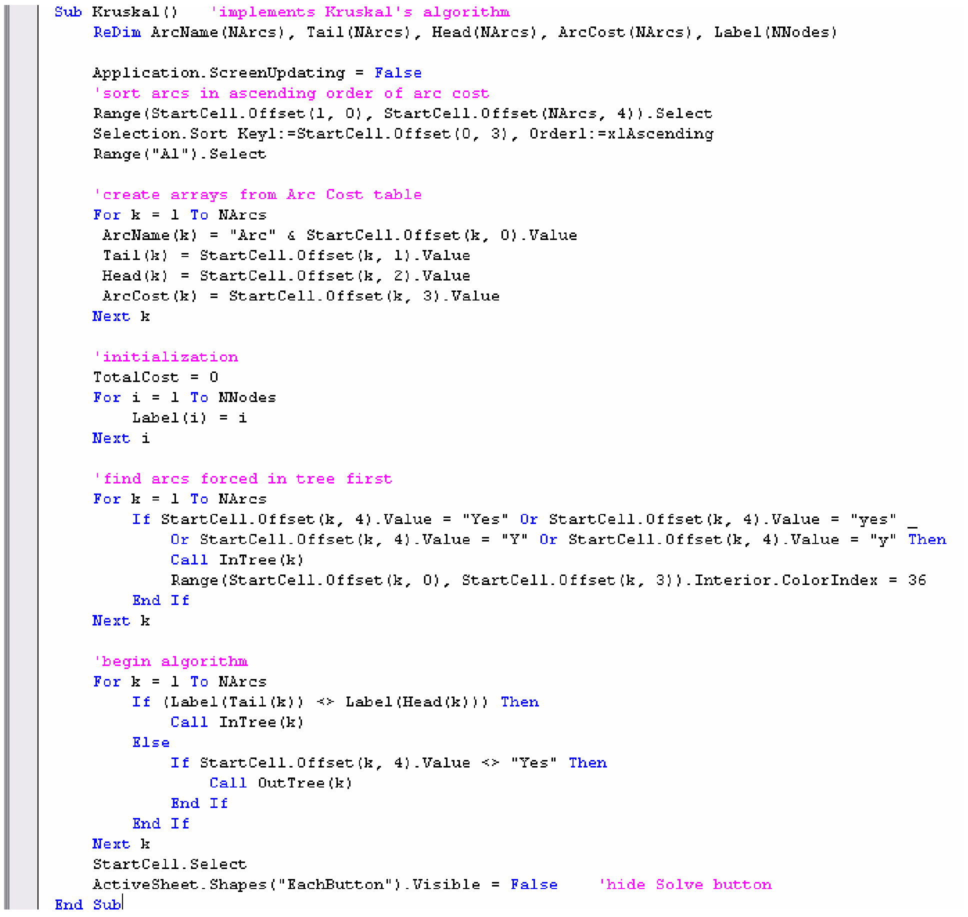
The procedures for the input form simply record the number of nodes and arcs (see Figure CS16.11). Some error checking is also performed.



```
Sub cmdCancel_Click()
    Worksheets("Welcome").Visible = True
    Worksheets("Network").Visible = False
    Unload Me
    End
End Sub
Sub cmdOK_Click()       'from Input Form
    Dim ctl As Control
    For Each ctl In Me.Controls
        If TypeName(ctl) = "TextBox" Then
            If IsNumeric(ctl.Value) = False Or ctl.Value = "0" Then
             MsgBox "Please fill in all values."
             ctl.SetFocus
             Exit Sub
            End If
        End If
    Next

    NNodes = txtNumNodes
    NArcs = txtNumArcs

    Unload Me
End Sub
```

Figure CS16.11 The input form procedures.

When the "Solve" button is pressed, the *Kruskal* procedure is called (see Figure CS16.12). This procedure implements and animates Kruskal's algorithm for finding the minimum spanning tree. The arc table is first sorted in ascending order of the arc costs. Then arrays are populated with the arc names, tail nodes and head nodes (or node pairs), and arc costs from the arc table. A labeling array is initialized; it will be used to check for cycles as the spanning tree is created.



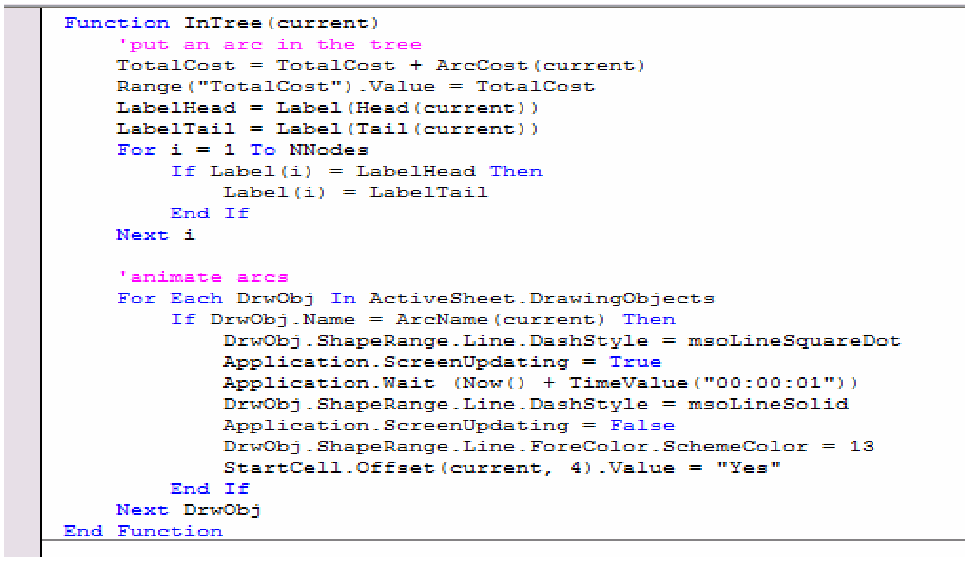
```
Sub Kruskal()   'implements Kruskal's algorithm
    ReDim ArcName(NArcs), Tail(NArcs), Head(NArcs), ArcCost(NArcs), Label(NNodes)

    Application.ScreenUpdating = False
    'sort arcs in ascending order of arc cost
    Range(StartCell.Offset(1, 0), StartCell.Offset(NArcs, 4)).Select
    Selection.Sort Key1:=StartCell.Offset(0, 3), Order1:=xlAscending
    Range("A1").Select

    'create arrays from Arc Cost table
    For k = 1 To NArcs
     ArcName(k) = "Arc" & StartCell.Offset(k, 0).Value
     Tail(k) = StartCell.Offset(k, 1).Value
     Head(k) = StartCell.Offset(k, 2).Value
     ArcCost(k) = StartCell.Offset(k, 3).Value
    Next k

    'initialization
    TotalCost = 0
    For i = 1 To NNodes
        Label(i) = i
    Next i

    'find arcs forced in tree first
    For k = 1 To NArcs
        If StartCell.Offset(k, 4).Value = "Yes" Or StartCell.Offset(k, 4).Value = "yes" _
            Or StartCell.Offset(k, 4).Value = "Y" Or StartCell.Offset(k, 4).Value = "y" Then
            Call InTree(k)
            Range(StartCell.Offset(k, 0), StartCell.Offset(k, 3)).Interior.ColorIndex = 36
        End If
    Next k

    'begin algorithm
    For k = 1 To NArcs
        If (Label(Tail(k)) <> Label(Head(k))) Then
            Call InTree(k)
        Else
            If StartCell.Offset(k, 4).Value <> "Yes" Then
                Call OutTree(k)
            End If
        End If
    Next k
    StartCell.Select
    ActiveSheet.Shapes("EachButton").Visible = False    'hide Solve button
End Sub
```

Figure CS16.12 The *Kruskal* procedure.

The algorithm then simply checks the label array for each arc in its sorted order to determine whether or not a cycle would be created if it were added to the spanning tree. If no cycle would be created then it is added to the tree by calling the *InTree* function. If a cycle would be created, then it cannot be put in the tree and the *OutTree* function is called. We will discuss later why we check for arcs which are forced in the spanning tree (see Section CS23.5).

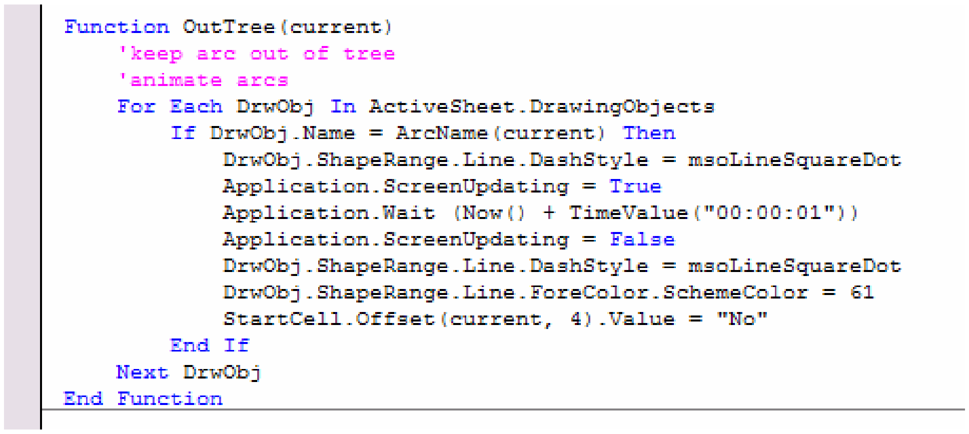
The *InTree* function updates the label array used in the algorithm and then animates the network (see Figure CS16.13). The animation changes the current arc format to dashed, pauses using the *Wait* method, and then changes the color of the arc to show that it is in the tree.

The *OutTree* function also animates the network (see Figure CS16.14). The animation changes the current arc format to dashed, pauses using the *Wait* method, and then changes the color of the arc to show that it is not in the tree.



```
Function InTree(current)
    'put an arc in the tree
    TotalCost = TotalCost + ArcCost(current)
    Range("TotalCost").Value = TotalCost
    LabelHead = Label(Head(current))
    LabelTail = Label(Tail(current))
    For i = 1 To NNodes
        If Label(i) = LabelHead Then
            Label(i) = LabelTail
        End If
    Next i

    'animate arcs
    For Each DrwObj In ActiveSheet.DrawingObjects
        If DrwObj.Name = ArcName(current) Then
            DrwObj.ShapeRange.Line.DashStyle = msoLineSquareDot
            Application.ScreenUpdating = True
            Application.Wait (Now() + TimeValue("00:00:01"))
            DrwObj.ShapeRange.Line.DashStyle = msoLineSolid
            Application.ScreenUpdating = False
            DrwObj.ShapeRange.Line.ForeColor.SchemeColor = 13
            StartCell.Offset(current, 4).Value = "Yes"
        End If
    Next DrwObj
End Function
```

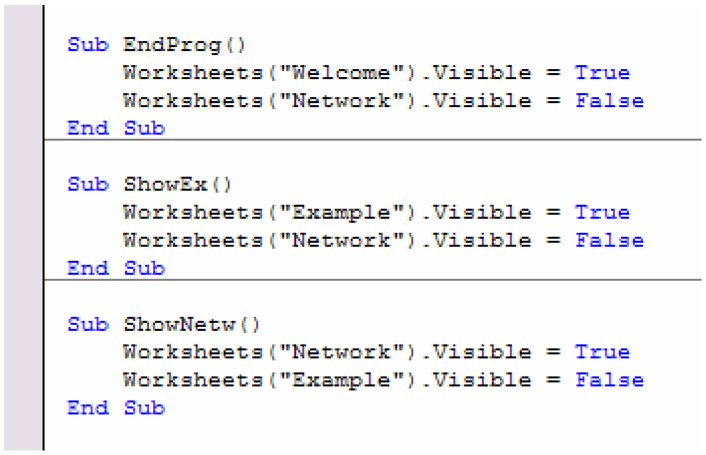
Figure CS16.13 The *InTree* function.



```
Function OutTree(current)
    'keep arc out of tree
    'animate arcs
    For Each DrwObj In ActiveSheet.DrawingObjects
        If DrwObj.Name = ArcName(current) Then
            DrwObj.ShapeRange.Line.DashStyle = msoLineSquareDot
            Application.ScreenUpdating = True
            Application.Wait (Now() + TimeValue("00:00:01"))
            Application.ScreenUpdating = False
            DrwObj.ShapeRange.Line.DashStyle = msoLineSquareDot
            DrwObj.ShapeRange.Line.ForeColor.SchemeColor = 61
            StartCell.Offset(current, 4).Value = "No"
        End If
    Next DrwObj
End Function
```

Figure CS16.14 The *OutTree* function.

The navigational procedures are used for the "End", "Show Example" and "Return to Network Sheet" buttons (see Figure CS16.15).



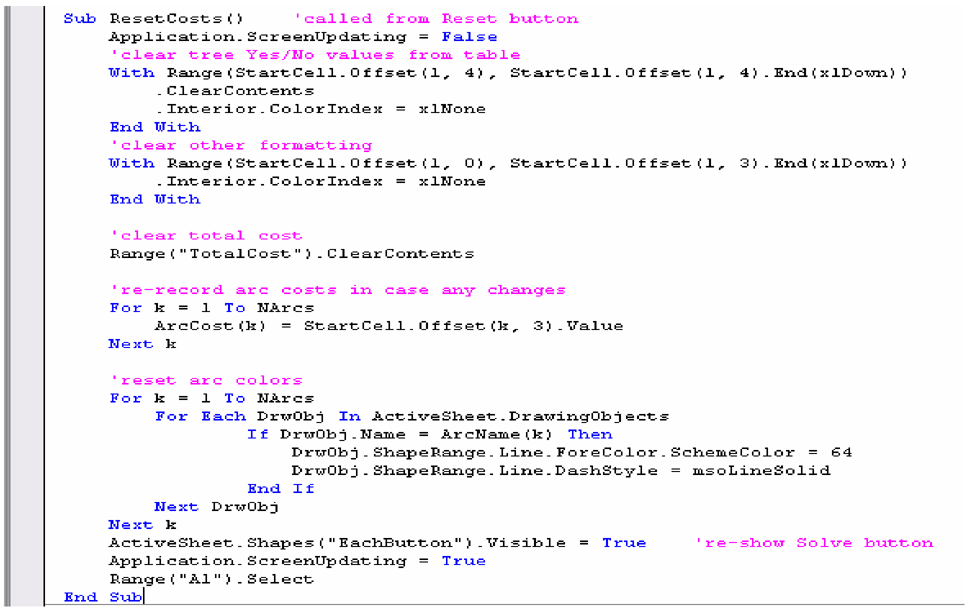
```
Sub EndProg()
    Worksheets("Welcome").Visible = True
    Worksheets("Network").Visible = False
End Sub

Sub ShowEx()
    Worksheets("Example").Visible = True
    Worksheets("Network").Visible = False
End Sub

Sub ShowNetw()
    Worksheets("Network").Visible = True
    Worksheets("Example").Visible = False
End Sub
```

Figure CS16.15 The Navigational procedures.

Summary		
	Main	Calls ClearPrevious and takes user to network sheet.
	Demo	Calls ClearPrevious, takes user to network sheet, and copies network from example sheet.
	ClearPrevious	Initializes variables and clears previous values and formatting.
	CreateTable	Creates arc table, shows input form, and performs some formatting.
	Input form procedures	Receives values for number of nodes and arcs in the user's network.
	Kruskal	Implements and animates Kruskal's algorithm.
	InTree function	Updates label array and animates putting arc in tree.
	OutTree function	Animates keeping arc out of tree.
	Navigational procedures	For "End", "Show Example" and "Return to Network Sheet" buttons

CS16.5 *Re-solve Options*

There are two main re-solve options for this application. Both options require the user to press the "Reset" button and then the "Solve" button on the network sheet. The "Reset" button is assigned to the *ResetCosts* procedure (see Figure CS16.16). This procedure clears the last column in the arc table which was filled by the algorithm. It also clears other formatting on the arc table and the value in the total cost cell. The arc costs are re-recorded and the network images are restored to their original formatting.

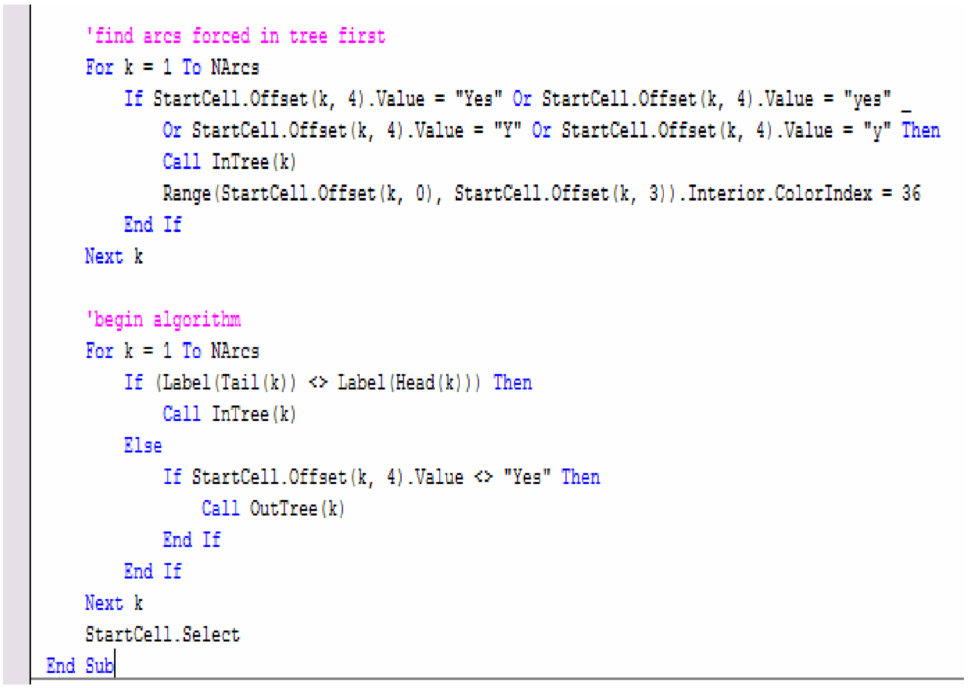


```
Sub ResetCosts()     'called from Reset button
    Application.ScreenUpdating = False
    'clear tree Yes/No values from table
    With Range(StartCell.Offset(1, 4), StartCell.Offset(1, 4).End(xlDown))
        .ClearContents
        .Interior.ColorIndex = xlNone
    End With
    'clear other formatting
    With Range(StartCell.Offset(1, 0), StartCell.Offset(1, 3).End(xlDown))
        .Interior.ColorIndex = xlNone
    End With

    'clear total cost
    Range("TotalCost").ClearContents

    're-record arc costs in case any changes
    For k = 1 To NArcs
        ArcCost(k) = StartCell.Offset(k, 3).Value
    Next k

    'reset arc colors
    For k = 1 To NArcs
        For Each DrwObj In ActiveSheet.DrawingObjects
            If DrwObj.Name = ArcName(k) Then
                DrwObj.ShapeRange.Line.ForeColor.SchemeColor = 64
                DrwObj.ShapeRange.Line.DashStyle = msoLineSolid
            End If
        Next DrwObj
    Next k
    ActiveSheet.Shapes("EachButton").Visible = True     're-show Solve button
    Application.ScreenUpdating = True
    Range("A1").Select
End Sub
```

Figure CS16.16 The *ResetCosts* procedure.

After pressing the “Reset” button and before pressing the “Solve” button to re-run Kruskal’s algorithm, the user can use either or both of the two re-solve options. The first option is for the user to modify the arc costs and see how it affects the minimum spanning tree. That is, if some arc costs are changed, when the algorithm is implemented and the arcs are sorted and cycles are checked, the resulting minimum spanning tree may differ from the original result.

The second option is for the user to force some arcs to be in the spanning tree. That is, suppose some constraints require a particular arc to be in the spanning tree regardless of its arc cost and node pairs. This re-solve option uses part of the code in the *Kruskal* procedure to check for these forced arcs before implementing the algorithm (see Figure CS16.17). The user can force the arcs in the tree by typing “Yes” (or “yes” or “Y” or “y”) in any row in the last column of the arc table.

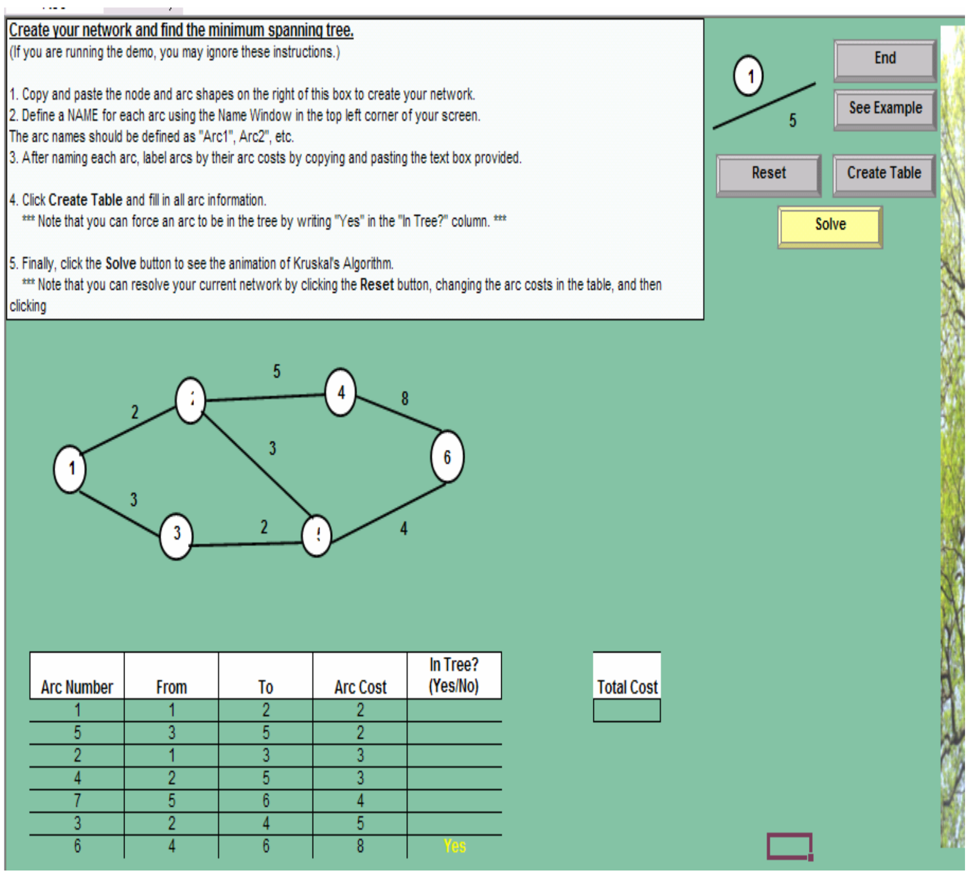


```
    'find arcs forced in tree first
    For k = 1 To NArcs
        If StartCell.Offset(k, 4).Value = "Yes" Or StartCell.Offset(k, 4).Value = "yes" _
            Or StartCell.Offset(k, 4).Value = "Y" Or StartCell.Offset(k, 4).Value = "y" Then
            Call InTree(k)
            Range(StartCell.Offset(k, 0), StartCell.Offset(k, 3)).Interior.ColorIndex = 36
        End If
    Next k

    'begin algorithm
    For k = 1 To NArcs
        If (Label(Tail(k)) <> Label(Head(k))) Then
            Call InTree(k)
        Else
            If StartCell.Offset(k, 4).Value <> "Yes" Then
                Call OutTree(k)
            End If
        End If
    Next k
    StartCell.Select
End Sub
```

Figure CS16.17 Code to capture forced arcs from *Kruskal* procedure.

For example, in Figure CS16.18, a minimum spanning tree is shown after the “Solve” button was pressed the first time; this is the original solution. As you can see, the arc with cost 8 has not been included in the minimum spanning tree. However, we can force this arc to be in the tree. To do so, we have typed “Yes” in the row for this arc in the last column of the arc table. We did this after we pressed the “Reset” button and before we press the “Solve” button again.

Once the “Solve” button is pressed this time, the *Kruskal* procedure will first identify this arc as a forced arc and put in the tree. It will then continue implementing the algorithm given that the label array has been updated for this first arc already. In Figure CS16.19, you can now see that this arc, with cost 8, has been included in the minimum spanning tree. The row for any arc forced in the tree is highlighted to inform the user that the algorithm did indeed force this arc in the tree.



Create your network and find the minimum spanning tree.
(If you are running the demo, you may ignore these instructions.)

1. Copy and paste the node and arc shapes on the right of this box to create your network.
2. Define a NAME for each arc using the Name Window in the top left corner of your screen.
The arc names should be defined as "Arc1", Arc2", etc.
3. After naming each arc, label arcs by their arc costs by copying and pasting the text box provided.

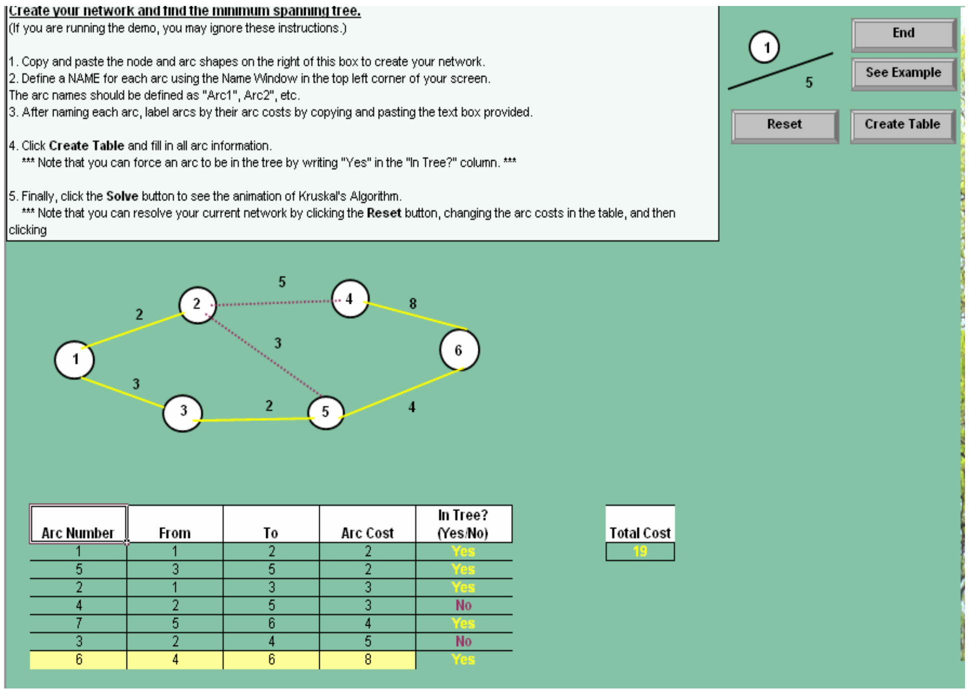
4. Click **Create Table** and fill in all arc information.
*** Note that you can force an arc to be in the tree by writing "Yes" in the "In Tree?" column. ***

5. Finally, click the **Solve** button to see the animation of Kruskal's Algorithm.
*** Note that you can resolve your current network by clicking the **Reset** button, changing the arc costs in the table, and then clicking

Arc Number	From	To	Arc Cost	In Tree? (Yes/No)
1	1	2	2	
5	3	5	2	
2	1	3	3	
4	2	5	3	
7	5	6	4	
3	2	4	5	
6	4	6	8	Yes

Total Cost

Figure CS16.18 The user can enter "Yes" in the table to force arcs in the tree.



Create your network and find the minimum spanning tree.
(If you are running the demo, you may ignore these instructions.)

1. Copy and paste the node and arc shapes on the right of this box to create your network.
2. Define a NAME for each arc using the Name Window in the top left corner of your screen.
The arc names should be defined as "Arc1", Arc2", etc.
3. After naming each arc, label arcs by their arc costs by copying and pasting the text box provided.

4. Click **Create Table** and fill in all arc information.
*** Note that you can force an arc to be in the tree by writing "Yes" in the "In Tree?" column. ***

5. Finally, click the **Solve** button to see the animation of Kruskal's Algorithm.
*** Note that you can resolve your current network by clicking the **Reset** button, changing the arc costs in the table, and then clicking

Arc Number	From	To	Arc Cost	In Tree? (Yes/No)
1	1	2	2	Yes
5	3	5	2	Yes
2	1	3	3	Yes
4	2	5	3	No
7	5	6	4	Yes
3	2	4	5	No
6	4	6	8	Yes

Total Cost
19

Figure CS16.19 The new solution with the forced arc.

Summary		
	"Reset" and "Solve" buttons	These buttons allow the user to user either or both of the re-solve options.
	Changing arc costs	The user can change arc costs and re-solve.
	Forcing arcs in tree	The user can force some arcs to be in the spanning tree and then re-solve.

CS16.6 *Summary*

- This case study allows the user to specify a network and animate Kruskal's Algorithm to identify a minimum spanning tree.
- We use three worksheets in this application: the welcome sheet, an example sheet, and the network sheet.
- For this application, we use navigational and functional buttons as well as one user form for the user interface.
- There are several procedures for this application which implement and animate Kruskal's algorithm.
- The user can re-solve the application by using the "Reset" and "Solve" buttons on the network sheet. They can change arc costs or force arcs to be in the spanning tree.

CS16.7 *Extensions*

- Allow user to also be able to change other network information before resolving. For example, you should be able to record not only new arc costs, but also new arcs and new nodes.
- Design an alternative user interface for all of the options available to the user on the network sheet.
- Add a procedure which automatically draws the network after the user enters the number of nodes and arcs.
- Modify this application to implement and animate Prim's algorithm for finding a minimum spanning tree. (Refer to Network Flows: Theory, Algorithm, and Applications by Ahuja, Magnanti, and Orlin.)
- Modify this application to implement and animate Sollin's algorithm for finding a minimum spanning tree. (Refer to Network Flows: Theory, Algorithm, and Applications by Ahuja, Magnanti, and Orlin.)